Tutorial 3

Resilient Communication Systems
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Channel models

Hints and Definitions

a. **Clarke's scattering model:** $y(t) = \int_0^{2\pi} f(\theta)\beta(\theta)x(2\pi f_c t - \tau_\theta(t))\, d\theta$, where $\beta(\theta)$, τ_θ and $f(\theta)$ are the attenuation for a scattering component impinging on the receiver under direction θ, the delay from the direction θ and PDF of θ, respectively.

b. **The one-ring model:** If we assume uniform power distribution and isotropic antenna gain pattern, i.e., the amplitudes $f(\theta) = \frac{1}{2\pi} \quad \theta \in (0, 2\pi)$ and $\beta(\theta) = \beta$, this models the situation when the scatterers are located in a ring around the mobile.

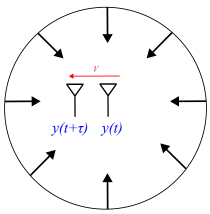


Figure 1: The one-ring model.

Problem 1

1. Based on Hint b, calculate the autocorrelation function $R_y(\tau)$ and the power spectral density $S(f)$ when receiver antenna moves with velocity v.
Hint: If $x(t) = \cos(2\pi f_c t)$ then according to the Fig. 1 first affect the Doppler impact on the received signal in different directions then calculate the autocorrelation.

2. Assuming that fading states separated by T_c are approximately uncorrelated, at what distance should we place a second antenna at the mobile to get an independently faded signal? *Hint:* How much distance does the mobile travel in T_c?

Problem 2

For uniform planar array (UPA) transmitter and receiver and with considering the following assumptions, compute $\mathbf{R}_{\text{rx}} = c\mathbb{E}\{\mathbf{H}\mathbf{H}^{\mathsf{H}}\}$ and $\mathbf{R}_{\text{tx}} = c\mathbb{E}\{\mathbf{H}^{\mathsf{H}}\mathbf{H}\}$ and also the factor c:

$$\mathbf{H} = \frac{1}{\sqrt{P}} \sum_{p=1}^{P} h_p \mathbf{a}_{\text{rx}}(\theta_p)\mathbf{b}_{\text{tx}}^{\mathsf{H}}(\phi_p), \tag{1}$$

where $\mathbf{R}_{\text{tx}} \in \mathbb{C}^{N_{\text{tx}} \times N_{\text{tx}}}$ and $\mathbf{R}_{\text{rx}} \in \mathbb{C}^{N_{\text{rx}} \times N_{\text{rx}}}$ denote the transmit and receive correlation matrices, respectively.

Assumptions:

1. The Tx and Rx arrays are deployed in an isotropic rich scattering environment where the multi-path components are uniformly distributed over the half-space in front of them.
2. $\frac{h_p}{\sqrt{P}}$ denotes the complex signal attenuation of the pth channel path. Moreover, $h_p \quad \forall p$, are i.i.d. RVs with mean zero and variance σ_c^2.

Hint: Since the UPA is deployed in an isotropic scattering environment, the multipath components are uniformly distributed over the half-space in front of it, which is characterized by the probability density function (PDF):

$$f(\theta,\phi) = \frac{\cos(\theta)}{2\pi}, \quad \theta \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right], \phi \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right].$$

References

[1] A. Goldsmith, *Wireless communications*. Cambridge university press, 2005.

[2] D. Tse and P. Viswanath, *Fundamentals of wireless communication*. Cambridge university press, 2005.

[3] E. Björnson and L. Sanguinetti, "Rayleigh fading modeling and channel hardening for reconfigurable intelligent surfaces," *IEEE Wireless Commun. Lett.*, vol. 10, no. 4, pp. 830–834, 2021.

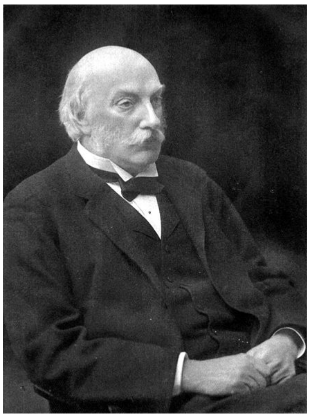
[4] V. Jamali, W. Ghanem, R. Schober, and H. V. Poor, "Impact of channel models on performance characterization of RIS-assisted wireless systems," in *Proc. European Conf. Ant. Prop. (EuCAP)*, Florence, Italy, 2023, pp. 1–5.

Wireless Future is a YouTube channel created for students interested in wireless communication and signal processing. Hosted by Professor Emil Björnson, a leading researcher in the field, the channel covers essential topics like Rayleigh fading, MIMO systems, and next-generation wireless technology (5G/6G). Professor Björnson explains complex ideas in a way that's easy to understand, often linking theory with practical examples. If you want to raise your knowledge in wireless communication studies, **Wireless Future** is a great resource to help you grasp the fundamentals and stay updated on the latest advancements in the field. You can start with the following video:
Rayleigh Fading Channels

Wireless Future

John William Strutt, 3rd Baron Rayleigh

John William Strutt, also known as Lord Rayleigh, was a British physicist who made groundbreaking discoveries about waves and electromagnetic theory. His research formed the basis of the Rayleigh fading model, used widely in communications. Rayleigh studied wave behaviors, especially scattering, which are key to the Rayleigh channel model. This model, which assumes no direct line of sight, is perfect for simulating urban mobile environments filled with obstacles that cause wave scattering. His work has greatly influenced telecommunications, particularly in wireless systems where managing multipath fading is essential.



John William Strutt
Born: 12 November 1842
Died: 30 June 1919 (aged 76)